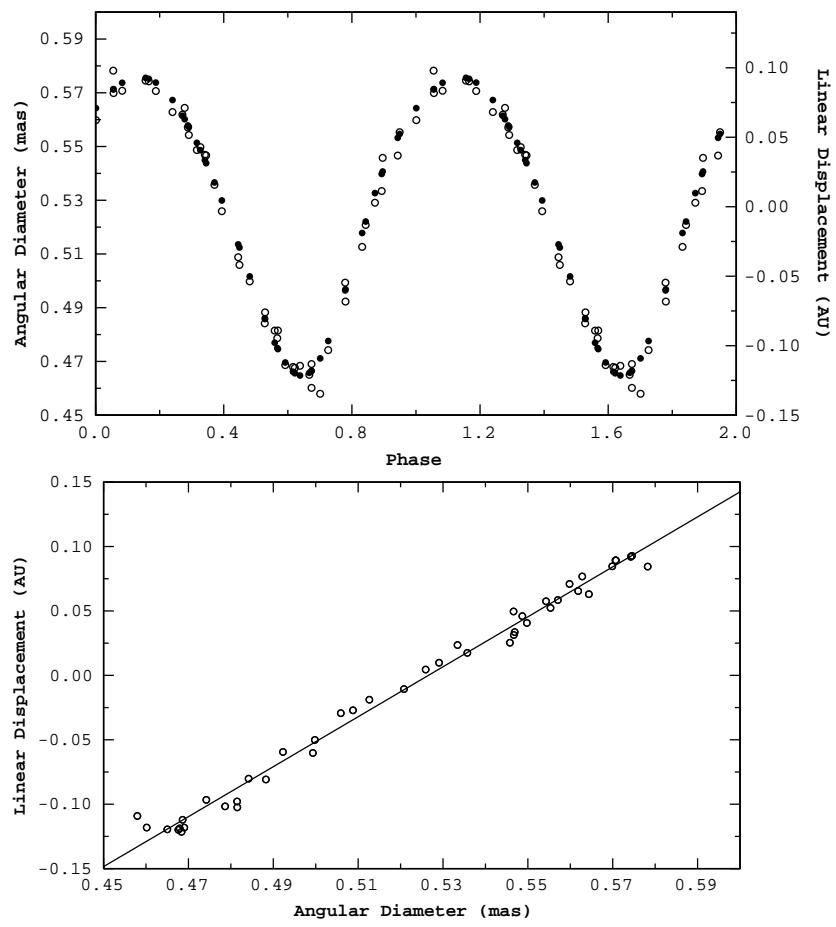


Table 1. Sources of adopted Radial Velocity and Optical Photometric Data

Cepheid	Radial Velocity Data	N	V, R data	N
EV Sct	Bersier et al. 1994 Metzger et al. 1991	40	Moffett & Barnes 1984 Berdnikov & Turner 1995	43
SZ Tau	Bersier et al. 1994	24	Moffett & Barnes 1984	35
QZ Nor	Coulson & Caldwell 1985b Metzger et al. 1992	16	Coulson & Caldwell 1985b Berdnikov & Turner 1995	49
CV Mon	Gieren et al. 1996 Metzger et al. 1992	47	Gieren et al. 1996 Berdnikov & Turner 1995	54
V Cen	Gieren 1981a	25	Gieren 1981b	35
BB Sgr	Gieren 1981a	24	Gieren 1981b Moffett & Barnes 1984	72
U Sgr	Mermilliod et al. 1987	42	Gieren 1981b Moffett & Barnes 1984	81
S Nor	Mermilliod et al. 1987 Bersier et al. 1994	30	Walraven et al. 1964 Breger 1970	61
V340 Nor	Mermilliod et al. 1987 Bersier et al. 1994 Metzger et al. 1992	41	Coulson & Caldwell 1985b Eggen 1983	30
VY Car	Coulson & Caldwell 1985a	60	Coulson & Caldwell 1985a	25
RZ Vel	Coulson & Caldwell 1985a	85	Coulson & Caldwell 1985a Berdnikov & Turner 1995	53
WZ Sgr	Coulson & Caldwell 1985a	21	Coulson & Caldwell 1985a Moffett & Barnes 1984	73
SW Vel	Coulson & Caldwell 1985a	60	Coulson & Caldwell 1985a	27
T Mon	Coulson 1983	50	Coulson & Caldwell 1985a Moffett & Barnes 1984	51
U Car	Coulson & Caldwell 1985a	51	Coulson & Caldwell 1985a Berdnikov & Turner 1995	51
SV Vul	Bersier et al. 1994	84	Berdnikov 1986, 1987	95



Abstract

We have obtained the radii and distances of 16 galactic Cepheids supposed to be members in open clusters or associations using the new optical and near-infrared calibrations of the surface brightness (Barnes-Evans) method given by Fouqué & Gieren (1997). We find excellent agreement of the radii and distances produced by both infrared techniques which use the V, V-K (K on the Carter system) and K, J-K magnitude-color combinations, respectively, and typical random errors as small as ~ 2 percent. We discuss in detail possible systematic errors in our infrared solutions and conclude that the typical total uncertainty of the infrared distance and radius of a Cepheid is about 3 percent in both infrared solutions, provided that the data are of excellent quality and that the amplitude of the color curve used in the solution is larger than ~ 0.3 mag. The optical V, V-R distance and radius of a given Cepheid can deviate by as much as ~ 30 percent from the infrared value, due to large systematic and random errors caused by microturbulence and gravity variations which affect the optical V-R color, but not the V-K and J-K colors, as shown by Laney & Stobie (1995).

We find excellent agreement of our infrared radii with the infrared radii derived by Laney & Stobie (1995) for these variables from an application of the maximum likelihood technique, which further increases our confidence that the total errors in our infrared solutions are not larger than ~ 3 percent. In an Appendix we discuss the relative advantages and disadvantages of our infrared surface brightness technique and the maximum likelihood technique. We compare the adopted infrared distances of the Cepheid variables to the ZAMS-fitting distances of their supposed host clusters and associations and find an unweighted mean value of the distance ratio of 1.02 ± 0.04 . A detailed discussion of the individual Cepheids shows that the uncertainty of the ZAMS-fitting distances varies considerably from cluster to cluster. We find clear evidence that four Cepheids are not cluster members (SZ Tau, T Mon, U Car and SV Vul) while we confirm cluster membership for V Cen and BB Sgr for which the former evidence for cluster membership was only weak. After rejection of non-members, we find a weighted mean distance ratio of 0.969 ± 0.014 , with a standard deviation of 0.05, which demonstrates that both distance indicators are accurate to better than 5%, including systematic errors, and that there is excellent agreement between both distance scales.

Subject headings: Stars: Cepheids - stars: distances - infrared: stars - distance scale - clusters: open

1. Introduction

Cepheid variables are crucial for the establishment of absolute distances to a sample of galaxies in the near field ($\sim \leq 30$ Mpc). These Cepheid-calibrated galaxies will then in turn serve to calibrate useful secondary distance indicators, like Supernovae Ia (Saha et al. 1996) which reach out to the far field of the Hubble flow, and from the observation of these objects in remote galaxies a reliable and accurate value of the Hubble constant will eventually be derived.

While HST makes it now possible to obtain distances to Virgo cluster galaxies based on their Cepheid variables which are accurate to 0.15 - 0.20 mag (Ferrarese et al. 1996), these are distances as measured *relative to the LMC*. The largest uncertainty in deriving the *absolute distances* to these galaxies still lies in our limited ability to measure the distances to local Cepheid variables, and in the corresponding uncertainty of the true distance to the LMC. In view of the enormous importance of the Cepheid process to establish a basic set of accurate distances to nearby galaxies, the improvement of the *local calibration of Cepheid distances* is more important than ever.

One of the fundamental techniques to measure the distance, and the radius of a Cepheid is the Barnes-Evans method (hereafter BE) which makes use of the observed variations in light, color and radial velocity of the variable. In this method, it is assumed that the surface brightness of the variable, at any given phase, can be accurately inferred from a measured color index at this phase - the degree to which this basic assumption is fulfilled critically determines both random and systematic errors in the radius and distance result. While in the past most applications of the Barnes-Evans method to Cepheid variables have been carried out in the optical spectral range, more recent work using near-infrared magnitudes and colors (e.g. Welch 1994) has shown that infrared colors are much better surface brightness indicators than optical ones. This has been demonstrated particularly convincingly in the work of Laney & Stobie (1995) who were able to trace down the physical causes for the inferiority of optical colors, which are the variable microturbulence and gravity during a Cepheid's pulsation cycle which invalidate, to a significant extent, the basic assumption of the Barnes-Evans method for optical colors while their effect on infrared colors is almost negligible. A further, important advantage in going to infrared colors is to avoid photometric contamination of Cepheid light and color curves by a companion - most Cepheid companions are blue stars which do affect optical, but not infrared colors.

Motivated by these results, Fouqué & Gieren (1997; hereafter Paper I) have provided new infrared calibrations of Barnes-Evans technique. In addition to using the V-K color index which was already used in a former study of Welch (1994), Paper I also provides a K, J-K version of the technique which makes only use of near-infrared magnitudes and colors. One particular strength of the new calibrations given in Paper I is that the zero point of the surface brightness- color relations was derived solely on the basis of accurate, interferometrically measured angular diameters of cool giant and supergiant stars, making the distance results independent of Cepheid model atmospheres or effective temperature scales which had to be used in the past. A first application of the method to the cluster Cepheid U Sgr had shown that there is a *dramatic increase in accuracy of the infrared solutions*, as compared to the optical one.

The purpose of this second paper is twofold. Firstly, we shall provide and discuss near-infrared radii and distances for many more Cepheid variables, and compare them to the optical solutions which we shall also derive. This will provide a clear picture regarding the relative strengths and weaknesses of the different variants of our technique. We shall also give a thorough discussion of the sources and magnitudes of the errors in our results, and we shall demonstrate that the near-infrared technique is able to produce radii and

distances of Cepheids with a typical accuracy of ± 3 percent. Secondly, we shall compare the infrared distances to the distance scale set by cluster ZAMS-fitting - to this end, we are doing our analyses in this paper on all Cepheids in open clusters and associations for which the necessary data are available in the literature. We shall discuss the Cepheid period-radius and most importantly, period-luminosity relation (in different passbands) in a forthcoming paper which will incorporate the solutions on a larger number of variables for an improved statistics of the results.

Our paper is organized as follows: in section 2, we present and discuss the radius and distance solutions; in section 3, we compare the infrared with the optical results; in section 4, we test the stability of our solutions to various factors and present an error budget; and in section 5, we compare our infrared distances to the cluster ZAMS-fitting distances. In an Appendix, we further discuss the relative merits of our technique as compared to the maximum likelihood approach used by Laney & Stobie (1995).

2. Radius and Distance Solutions

2.1. The Sample of Galactic Cluster Cepheids and adopted Data

Laney & Stobie (1992) have obtained extensive near-infrared photometry of southern Cepheids on the Carter system, including all known or suspected cluster Cepheids south of $\delta = +30^\circ$, which represent about 75 percent of all galactic Cepheids in open clusters and associations (Feast & Walker 1987). For this sample of cluster Cepheids, we have conducted a literature survey to determine the highest-quality sources of the other data needed in our analyses, namely radial velocity curves, V light curves and V-R color curves. In this process it became clear that for a few of the cluster Cepheids some of the needed data sets, mostly the radial velocity curves, were not of sufficient quality for the analysis of this paper. Our choice in these (few) cases was to omit these stars in order to maintain a high and approximately uniform quality in the adopted data for the selected Cepheids. This defined a final set of 16 Cepheids adopted for this study. The sources of the adopted radial velocity and V,R photometric observations are given in Table 1. For all of these Cepheids, the near-infrared J and K data were taken from Laney & Stobie (1992), which ensures homogeneity in the adopted infrared photometry.

An important next step is the choice of the correct pulsation periods for the variables which ensures a correct phase match between data sets obtained at different epochs. Rather than to conduct a new analysis of the periods, we have relied on the work of Laney & Stobie (1992) and have adopted their periods for all variables. The results in section 2.2 will show that for most of the Cepheids, the adopted periods lead to excellent phase matches between data sets spaced by many pulsation cycles, but there are a few cases where the period values might be slightly improved, or where the period itself is variable (see section 3.3.). Another choice we have to make refers to the color excesses of the variables. We have chosen to adopt the excesses given by Fernie (1990) which are usually mean values from many individual

determinations, and have the advantage of having been derived on a homogeneous system. Fortunately, and this being one of the particular strengths of the BE method, the choice of the adopted color excesses is not critical for the radius and distance solutions (see section 4.1.). A final choice we have to make in our solutions refers to the p-factor which converts the observed radial into the pulsational velocities of the stellar surface. We adopt here the values which come from the formula given in Gieren, Barnes & Moffett (1993) which result in values of the p-factor which depend slightly on period. There is a possibility that the p-factor is not constant for a given Cepheid but shows some variation with phase (Sabbey et al. 1995); our results allow to conclude that such a variation is probably not important for the BE analysis for most of the Cepheids in our sample (see section 4.6.). In Table 2, we provide the values for periods, reddenings and p-factors we have adopted for this study. We also provide information on the uncertainties of the adopted reddenings, which were calculated as explained in the Appendix.

2.2. Optical and Near-Infrared Radius and Distance Solutions

In Paper I, we have derived expressions for the angular diameter of a Cepheid variable in terms of a de-reddened magnitude and color. These expressions are:

$$\theta = 10^{0.5474 - 0.2V_o + 0.760(V-R)_o} \quad (1)$$

$$\theta = 10^{0.5474 - 0.2V_o + 0.262(V-K)_o} \quad (2)$$

$$\theta = 10^{0.5474 - 0.2K_o + 0.220(J-K)_o} \quad (3)$$

Equ. (1) represents the optical calibration in terms of the V, V-R_J magnitude/color combination, whereas equ. (2) and (3) represent the infrared calibrations in terms of V, V-K and K, J-K. For each of the Cepheid variables, we have calculated the angular diameters from each of these equations using the photometric data from the sources given in Table 1, and correcting the data sets for possible shifts in phase and photometric zero point in cases where different data sets were combined. In the case of the optical V-R colors, those data sets obtained on the Cousins system were transformed to the Johnson system following the precepts of Gieren (1984) and allowing a variable zero point in the transformation formula to achieve best agreement with the colors observed on the Johnson system in each case. In order to obtain the V-K color curves of the variables from observations which in no case were obtained contemporaneously, we adopted the procedure to fit Fourier series to the K curves from which the values were taken which corresponded to the phases of the actual V observations.

The angular diameters were then combined with the linear displacements which were calculated from the integrated radial velocity curves. In order to perform the integration, we first fitted Fourier series of appropriate orders to the observed radial velocity curves of each star, with the criterion to obtain good fits of the observed structures, without overfitting the curves, i.e. introducing artificial undulation in the fitted curves. The linear displacements were then calculated at the phases of the photometric observations. The radii and distances were obtained from least-squares solutions of the relation

$$D_0 + \Delta D = 10^{-3} d \theta \quad (4)$$

where D_0 is the mean linear diameter of the Cepheid, ΔD the displacement from the mean (both in AU), d the distance (in pc), and θ the angular diameter at this particular phase (in mas). Before doing the solutions, we plotted the linear and angular diameter curves of each of the Cepheids to check for the correct phase match of both curves. In most cases, the curves matched very well and there was no need to apply a shift to the curves to bring them into phase agreement, but in some cases (and particularly in the V, V-R solutions) a small phase shift was applied.

In Table 3, we give the radius and distance results we obtained for each Cepheid in each of the three magnitude/color combinations, together with the corresponding standard deviations. A discussion of the errors involved in the determination of the linear displacements and angular diameters (see Appendix) shows that in all cases the angular diameters bear larger relative uncertainties than the linear displacements; under these circumstances, the correct fit to adopt in the least-squares solutions of equ. (4) is the *inverse* fit (see Appendix). In order to show the sensitivity of our radius and distance results to the adopted fit, Table 3 also gives the differences of the adopted inverse to the corresponding direct fit (which assumes all errors in the linear displacements), in units of the standard deviation σ of the inverse solution. As can be seen, the difference is negligible in the infrared solutions while it can be important in the optical solution. We discuss this further in section 4.3. Column 7 of Table 3 gives the adopted phase shifts between displacement and angular diameter curves, and the last column of Table 3 contains information about the number of photometric points used in the solution. Ideally, the phase shift between the angular diameter and the linear displacement curve should be the same in the different solutions because the star is pulsating with a fixed period. In a few cases, however, the best match occurs for slightly different phase shifts for the same Cepheid. This is probably a consequence of the observational scatter, but it could also reflect a deeper problem with the phase shifts. In any case, a slightly different phase shift as adopted by us for the infrared solutions of some of the stars in Table 3 does not have any important practical effect on the resulting radius and distance values because the infrared solutions are very insensitive to the adopted phase shifts (see discussion in section 4.3.).

In Figures 1-3 we show, as a typical example of our solutions, the variations of linear displacements and angular diameters, and the plots of linear vs. angular diameter (from which the distance and radius is derived), for the Cepheid VY Car as obtained from the 3 different magnitude/color combinations.

The values of the errors in Table 3 confirm the trend seen in Paper I in the results for U Sgr, namely that the uncertainties of both infrared solutions are generally much smaller than the one of the V, V-R solution, especially for the shorter-period stars. Typical values for the uncertainties of our radius and distance results are 5-10 percent in V, V-R, 2-3 percent in K, J-K, and 1-2 percent in V, V-K. In Figure 4, we have plotted the uncertainties of the radii and distances (normalized to a fixed number of 30 photometric observations per star) against the amplitudes of the color curves used in the solutions. As expected, there is a strong correlation: in the K, J-K solutions, there is sharp increase in the errors if the amplitude of the color curve drops below 0.1 mag. This is expected because in this case the

variation of the color term in equ. (3) is in the order of only 0.01 mag, i.e. in the order of the photometric uncertainties. On the other hand, at an amplitude of only 0.13 mag in J-K an accuracy of $\pm 4\%$ can already be reached, and for the long-period Cepheids which have typical amplitudes of 0.3 mag the accuracy increases to $\sim 1\%$ if the data are good enough and the extrema of the color curve are sampled by the observations. The situation in the V, V-K solutions is even more favorable because the amplitudes of Cepheid V-K color curves are typically 3 times larger than those of their J-K curves; Figure 4 shows that even for the lowest-amplitude Cepheids of our sample, a radius and distance accuracy of $\sim 2\%$ is easily obtainable, if the data used in the solutions are of adequate quality. For this reason, we can trust the V, V-K radius and distance values of the three shortest-period, low amplitude Cepheids in our sample, EV Sct, SZ Tau and QZ Nor (but see the notes in section 2.3), while we cannot place much confidence in the values coming from the other two magnitude/color combinations.

2.3. Notes on Individual Cepheids

In this section, we give notes on some of the stars which are important for our analysis.

EV Sct: The real uncertainty of the V-K radius and distance is probably higher than the error given in Table 3, due to the small number of K observations (12) available for the analysis. EV Sct is likely to be an overtone pulsator, but this does not affect its radius and distance derived from the Barnes-Evans technique.

SZ Tau: The period is known to be variable (Szabados 1977) which may have caused a phase mismatch between the V and K curves, and thus a systematic error in the V-K solution (see section 4.4). As EV Sct, SZ Tau is likely to be a first overtone pulsator.

CV Mon: There are considerable differences among published V light curves which are likely to be due to two nearby companion stars at 10" and 14" (Evans & Udalski 1994) which have or have not been included in the aperture photometries. The data used by us in this paper are not affected by this problem.

BB Sgr: This Cepheid is likely to be a spectroscopic binary (Barnes, Moffett & Slovak 1988). We have used for our analysis the radial velocity obtained by Gieren (1981 a) during only two consecutive pulsation cycles during which a possible change of the γ velocity of the Cepheid must be negligibly small, thus not affecting our analysis.

V340 Nor: The only published V data for this variable of Eggen (1983) and Coulson & Caldwell (1985 b) show unusual scatter for a Cepheid of this magnitude, affecting the accuracy of our V, V-R and V, V-K solutions. This is probably due to the position of the variable close to the center of NGC 6067 which makes accurate aperture photometry difficult. It would be very desirable to obtain a modern V light curve of this Cepheid from CCD observations and PSF fitting techniques.

T Mon: This Cepheid is known to be a spectroscopic binary (Gieren 1989). To avoid any problem with the variable γ velocity of the star, we have adopted the pulsational radial velocity curve given by Coulson (1983) which is freed from the orbital motion effect.

SV Vul: The period of this very long-period Cepheid is changing (Bersier et al. 1994).

However, we find that the period value adopted in this paper gives a satisfactory representation of all the radial velocity data of Bersier et al., and also defines light curves of reasonably low scatter. Nevertheless, there is a possibility of a phase mismatch between V and K, and thus of a systematic error, in the V-K solutions. Also, it appears likely that for a Cepheid as extended and luminous as SV Vul one or more of the assumptions of the Barnes-Evans method might fail, affecting the derived radius and distance in a systematic way. This possibility should be borne in mind when interpreting the radius and distance data of the most luminous Cepheids.

3. Comparison of the Near-Infrared to the Optical Solutions

3.1. Comparison of V, V-K to K, J-K Radii and Distances

In Figure 5, we have plotted the ratio of the Cepheid radii as derived from the V, V-K and K, J-K magnitude/color combinations against period. For most of the Cepheids, the agreement of the radius values from the two calibrations is excellent, better than about 4%. Omitting the 3 shortest-period Cepheids of our sample whose K, J-K radii cannot be trusted for the reasons given above, the mean ratio of the radii of the remaining 13 stars is 0.98 ± 0.012 . There is clearly no trend of this ratio with period. We thus conclude that there is no significant systematic difference between the V, V-K and K, J-K radius solutions.

In Figure 6, we have plotted the distance ratios vs. period. The plot looks very similar to the corresponding radius plot. The mean ratio of the distances is 0.97 ± 0.013 , and there is no trend with period either. The mean offset of 3 percent between the two solutions may be marginally significant, in view of the very small uncertainty of this result, but at this point we prefer to conclude that there is no significant difference between the distances as derived from the V, V-K and K, J-K versions of the method, either. A small difference in the order of 3% may however turn out to be real when we have better statistics, which will be the case in Paper III.

The important conclusion from this comparison is that there is an excellent level of agreement between individual radii and distances from both methods, without any significant or systematic (with period) offset between the results. This is an extremely important result which indicates that the low errors found in the solutions are real and that systematic uncertainties in both methods are below the $\sim 3\%$ level (but see section 4.6). *As a consequence, we adopt as our final radius and distance values for each Cepheid the weighted mean of the V, V-K and K, J-K solutions.* However, there are the following exceptions: as discussed above, the K, J-K solutions for the 3 shortest-period, low-amplitude Cepheids EV Sct, SZ Tau and QZ Nor are not reliable, and we therefore adopt for these stars the V, V-K solutions. Furthermore, in the V, V-K solutions for the 3 long-period Cepheids T Mon, U Car and SV Vul there are small but systematic deviations in the shapes of the linear displacement and angular diameter curves, which are *not* seen in the K, J-K solutions. We show this for U Car in Figure 7. The most likely cause for this effect is a small phase mismatch between the V

and K curves of these Cepheids, due to slightly incorrect period values or variable periods, as in the case of SV Vul, but the effect may also be due to an increasing departure from the basic assumptions of the Barnes-Evans methods shown by the longest-period stars which manifests itself in a more pronounced way in the V, V-K than in the K, J-K solutions. We therefore feel that it is the best choice to adopt the pure infrared solutions for these stars. In practice, this is not an important issue because the two solutions are very similar for U Car and SV Vul (there is, however, a larger discrepancy for T Mon for which the phase mismatch may be more serious).

The final, adopted radius and distance values for the Cepheids are given in columns 3 and 7 of Tab. 4.

3.2. Comparison of the Infrared to the Optical Solutions

In Figures 8 and 9 we compare the optical V, V-R_J radii and distances to the adopted values from the infrared solutions. Two effects can be immediately seen in these plots: firstly, both the optical radii and distances are on average clearly larger than their infrared counterparts, and secondly, there is a very significant scatter in these plots. The mean values for the ratios R(optical) / R (infrared) and d(optical) / d(infrared) are 1.13 ± 0.05 and 1.16 ± 0.06 , respectively. Disregarding the 3 shortest-period Cepheids for which the V, V-R_J solutions have very large random errors due to the very small amplitudes of the V-R color curve, the data in Figs. 8 and 9 demonstrate that for a given Cepheid with very good observational data, the deviation of its optical radius and distance from its infrared counterpart can be up to about 30 percent in both directions. This means that the effect of variable microturbulence and gravity on the optical V-R color does not only introduce large random, but also large systematic errors. Our impression on the basis of the few published data is that these systematic deviations might be mostly correlated with the mean microturbulence of a Cepheid, which can vary considerably from one Cepheid to another, even for stars of very similar pulsation period; this hypothesis could be tested with good spectroscopic microturbulence determinations.

As a conclusion, our solutions confirm the conclusions of Laney & Stobie (1995) about the lack of reliability of the optical radii in individual cases (the behavior of the optical radii of a large sample of Cepheids will be discussed in our next paper). This is, of course, the reason why we adopt the infrared solutions as our final values, which are obviously not plagued by the problems of the optical solutions.

3.3. Comparison of the Infrared Radii to published Work

In columns 5 and 6 of Table 4, we compare our infrared radii to the values found by Laney & Stobie (1995) from the corresponding magnitude/color combination using the maximum likelihood technique, and to the values obtained by Gieren, Barnes & Moffett (1989) from their calibration of the optical V, V-R_J surface brightness method. Since the Laney & Stobie radii were obtained using a p-factor of 1.36 for all Cepheids, we have normalized our radii to this p-factor in the comparison. Omitting again the 3 shortest-period Cepheids from the

comparison (which have uncertain maximum likelihood radii for the same reason as in our analysis), the mean value of the ratio $R(\text{LS}) / R(\text{this paper})$ is 0.98 ± 0.03 , which means that *the radii from both sources agree within a very small uncertainty*. In Figure 10 we have plotted the radius ratios against period, and it can be seen that there is no significant trend with period. The most significant (20%) deviation for an individual star occurs for CV Mon; this may be related to the fact that we were able to use, in this study, a much improved radial velocity curve as compared to the one available to Laney and Stobie. If we omit CV Mon in the comparison, the mean ratio becomes 0.965 ± 0.013 , indicating a marginal possibility that the Laney & Stobie radii are on average 3% smaller than our radii, which is still very close to our result. We therefore conclude that *the fact that our infrared radii do agree so well with the ones obtained by Laney & Stobie who have derived them using quite a different approach (albeit sharing many of the data we used in our study, in particular the near-infrared photometry) lends further strong support to the idea that the infrared radii are very accurate, at the $\pm 3\%$ level.*

The Gieren, Barnes & Moffett (1989) radii are on average 11 percent larger than our adopted infrared radii, a number which shows that they basically do agree with the V, V-R_J radii derived in this study. This seems surprising given the fact that our new calibration of the optical surface brightness relation (equ. 26 in Paper I) tends to make the radii smaller. However, Gieren et al. used *direct* least squares fits for most of their Cepheids which yield smaller Cepheid radii than the inverse fits we have used in this paper, and both effects seem to cancel, to a first approximation.

4. Stability of the Radius and Distance Solutions

In order to establish the influence of various possible sources of error on our radius and distance results, we have carried out a series of tests whose results will be described in the following sections.

4.1. Absorption Corrections

Due to the structure of equations 1-3 for the Cepheid angular diameter, it is immediately seen that an error in the color excess used to correct the observed magnitude and color for interstellar absorption and reddening will tend to cancel out; for instance, if the value of E(B-V) used in the calculation is too large, the absorption-corrected magnitude will be too bright, but the intrinsic color index comes out too blue, and both errors will tend to cancel. Our numerical tests show that the sensitivity of the optical V, V-R and the infrared V, V-K methods to errors in the adopted color excess are very similar: a change of $\Delta E(\text{B-V}) = 0.15$, which corresponds to a change in the adopted value of Av of about 0.5 mag, produces a $\sim 3\%$ systematic change in the distance and leaves the radius virtually unchanged. The K, J-K version of the method is even more insensitive to the adopted reddening corrections: the same 0.15 mag change in E(B-V) produces only a 1.4% change in the distance while leaving the radius completely unaffected.

Since the typical uncertainty of the color excesses of the Cepheids given in Table 2 is about 0.04 mag, any systematic errors in the radii and distances due to uncertain reddening corrections will be below the 1 percent level, for both the optical and near-infrared versions of the method. *Absorption corrections are thus not a source of concern in the method.*

4.2. Direct or Inverse Least-Squares Fits?

A discussion of the errors in the linear displacements and angular diameters (see Appendix) shows that the correct fit to adopt in our solutions is the *inverse* least-squares fit. However, the data in columns 4 and 6 of Table 3 show that in both near-infrared solutions, the choice of the fit for those Cepheids with good observational data is unproblematic: the difference between the extreme solutions (direct and inverse fits) is always smaller than one standard deviation, in these cases, so it *doesn't matter how the fit is taken in the near-infrared solutions*. This is, of course, another way of saying that the random errors in the infrared solutions are very small. A consequence of this is that we are not dependent on having photometric data covering the complete pulsation cycle of the variable: solutions using data of ascending and descending branches of the light curve *alone*, for instance, lead to identical radius and distance values for the Cepheid.

The data in Table 3 show that this is generally not true for the optical solutions. Here, the difference between the direct and the inverse solution can be larger than 3σ (although for most stars it is less than this, especially for the longer-period Cepheids - see the data in Table 3), so the choice of which fit to adopt is a matter of concern in the optical solutions. This choice has to be made for each star on the basis of the quality and quantity of the observational data available for the analysis, which determine the relative errors in the linear and angular diameters, and taking into account the possible systematic errors in the angular diameters introduced by microturbulence and gravity variations. Therefore, the choice can be *different* for different stars.

Obviously, the insensitivity of the radii and distances derived with the near-infrared methods to the choice of the way the fits are taken is one of the great advantages of the infrared versions of the method over the optical one.

4.3. Phase Mismatch between Photometric and Radial Velocity Curves

It has been known for a long time that the correct phase linking between the radial velocity curve and the photometric curves in a BE analysis must be done with high precision to avoid significant systematic errors. We are sensitive to this source of error because in most cases, the radial velocity curve and the photometric curves of a given variable were not measured contemporaneously, and the periods are not known with perfect accuracy.

Figure 11 shows the typical dependence of the derived distance of a Cepheid on the phase mismatch between the radial velocity and photometric curves we find, in this case for the Cepheid U Sgr in the near-infrared K, J-K solution. Within a range of ± 0.05 in the phase mismatch, the typical change in the radius and distance value of a Cepheid is about *1% per phase mismatch of 0.01*. Our method is therefore much less sensitive to this phase mismatch

than the maximum likelihood solutions of Laney & Stobie (1995) who find that their radius values change by a worrisome 5% for a phase mismatch of 0.01. Figure 11 also shows the variation of the relative accuracy of the radius and distance result with the misalignment in phase between radial velocities and photometry; it is seen that there is a fairly pronounced minimum in the error if the phase alignment is correctly done. We can therefore use this criterion to help establish the correct phase relation between the radial velocity curve and the photometric curves (although there may be a possibility that a part of the observed phase shift is due to some intrinsic cause which we do not yet fully understand).

Our experience shows that in all cases we can achieve the correct phase alignment to within ± 0.01 using a) a visual alignment of the linear displacement and angular diameter curves, and b) the criterion of a minimum dispersion in the solution. This is at least true for the *near-infrared solutions*, due to the very low scatter in the angular diameters; for the much noisier optical solutions, we might have failed to do the phase alignment with this 1% precision for some of the stars. Also, the correct phase alignment is hampered in the optical solutions by the systematic differences in shape between the linear displacement and angular diameter curves which are present for nearly all Cepheids, but which are not seen in the infrared solutions (at least in the K, J-K solution; see section 4.4). We can therefore state that for the infrared solutions, the systematic errors we expect due to the phase alignment problem between the radial velocities and the photometry are very likely to be below the 1% level for most of the stars.

4.4. Phase Mismatch between V and K Curves in the V, V-K Solution

Another possible source of systematic error affecting the *V, V-K solutions* is a possible misalignment of the V and K light curves which in all instances have been obtained at different epochs, for the Cepheids in our sample. We must therefore check for the effect such a phase mismatch between V and K has on the resulting radius and distance. We did this by introducing, as in the case of the velocity curves and photometry discussed in the previous section, artificial phase shifts between the V and K light curves, which leads to a *distortion of the resulting V-K color curve* which is used in the radius and distance solution. Our tests show that the *change in the radius and distance is typically about 5% for a phase mismatch between V and K of 0.01*. This kind of phase mismatch (affecting only the V, V-K solution) has therefore a much more serious effect on the solutions than a possible phase mismatch between the radial velocities and photometric curves discussed in the previous section.

As a consequence of an increasing phase mismatch between V and K, the *amplitude* of the resulting V-K color curve, and the *phase of bluest V-K color* change systematically. We find that within a phase mismatch interval from -0.05 to +0.05 the *amplitude ratio A(V-K) / A(V)* decreases typically by 25%, while at the same time the *phase shift between the maxima of the V and V-K curves* varies from +0.08 to -0.04. Since there seems to be little intrinsic variation from Cepheid to Cepheid in both, the (V-K)/V amplitude ratio and the phase relation between the maxima of both curves, the observed amplitude ratio and phase shift provide in principle a useful criterion to deduce a phase mismatch between the V and K

curves, especially in cases where *both* values deviate from the normal values in the expected sense. However, due to the remaining intrinsic spread in these quantities from Cepheid to Cepheid these criteria are probably not too safe, and the only way to ensure a correct phase match between V and K is to have very accurate periods, or ideally to use contemporaneous or near-contemporaneous observations in both passbands.

As a conclusion, the correct phase alignment between the V and K curves is critical in the method and must be done with great care. The comparison of the V, V-K radii and distances to the K, J-K solutions and the excellent agreement found in section 3.1. demonstrates, however, that we have obviously succeeded to keep the errors from this source low, typically below the 2% level. It is likely that most of the remaining scatter among the two near-infrared solutions seen in Figs. 5 and 6 is due to small errors in the phase alignment between the V and K curves of the variables.

4.5. Uncertainties in the Calibrations of the Surface Brightness - Color Relations

An obvious source of systematic uncertainty comes from possible errors in the zero point and slopes of the three different surface brightness - color relations from which the Cepheid angular diameters are deduced. Our discussion in Paper I shows that the zero point (common to the three relations) is accurate to ± 0.003 while the uncertainties in the slope values are different for the different calibrations. We recall that the zero point rests solely on interferometrically measured angular diameters of cool giants and supergiants while the slopes were derived from a sample of Cepheids using Thompson's method. We have further shown in paper I that there is no detectable difference between the various surface brightness-color relations defined by the giants and supergiants of our calibrating sample. There is a possibility that stable supergiants have somewhat different colors from the supergiants with pulsating atmospheres, but the very good agreement of the slopes derived for the Cepheids with Thompson's method with those observed for the stable supergiants gives us a lot of confidence that Cepheids are obeying surface brightness-color relations which are basically indistinguishable from those obeyed by the stable supergiants and giants.

Our tests show that in the infrared methods, a variation of the *zero point* by 1σ causes a change in the distance of 2.4% and 2.2%, respectively, for the K, J-K and V, V-K versions of the method. The radii do not change because they do not depend on the zero point of the surface brightness-color relation. In the optical V, V-R method, the change in the distance is very small, only 0.2%. A change in the respective *slope* values by 1σ causes changes in the distance and radius of 2.3% and 2.8% in the near-infrared K, J-K and V, V-K methods, respectively, while the change in distance and radius in the optical method is 1.7%. These results show that while these errors are still important for the application of the different variants of the method, the effort we have made in Paper I to reduce the uncertainties in the calibrations of the surface brightness-color relations has been *crucial* to achieve the current low level of systematic uncertainties in the method (see Table 5).

4.6. The Conversion Factor from Radial to Pulsational Velocity

In each of our solutions, both the radius and distance are directly proportional to p , the factor adopted to convert the observed radial to pulsational velocities of the stellar surface. While most studies in the past have used constant p -factors with values between 1.31 (Parsons 1972) and 1.41 (Getting 1934), modern values over the last decade have clustered near $p=1.36$. The value to adopt has also a small dependence on the way the radial velocities are measured. An improvement came in the study of Gieren, Barnes & Moffett (1989) who introduced a period-dependent p -factor (but constant for a given Cepheid) based on a formula which was derived from the Cepheid models of Hindsley & Bell (1989). More recently, Sasselov & Karovska (1994) and Sabbe et al. (1995) have advocated the use of a p -factor which varies with phase during the pulsation of a Cepheid. This conclusion was based on non-LTE hydrodynamic models of Cepheid atmospheres in conjunction with high-resolution optical and near-infrared spectra for a few bright Cepheids. However, the effect of a variable p -factor is not seen in LTE calculations, and to date there remains some doubt about the reality and the true size of this effect.

Our observational results in this paper, i.e. *the near-perfect agreement in the shapes of the linear displacement and angular diameter curves* seen for nearly all Cepheids in the infrared solutions (especially in the K, J-K solutions where we have not the phasing problem discussed in section 4.4) *lend strong support to the conclusion that $p=\text{constant}$ must be a very good approximation for most Cepheids in our sample*, independent of their periods. Otherwise, systematic deviations in the shapes of the linear displacement curves vs. the angular diameter curves should show up, due to the distortions produced in the linear displacement curve by the (wrong) assumption that p is constant over the pulsation cycle. There are a few Cepheids where such small deviations are indeed observed; however, since we see them in the V, V-K solution, but *not* in the K, J-K solution, their origin is almost certainly in a slight phase mismatch between V and K, and not in neglecting a significant variation in p over the pulsation cycle.

From a comparison of published modern values of the p -factor (constant for a given Cepheid) we conclude that a realistic current uncertainty in the radius and distance from any BE technique due to this factor is $\sim \pm 2.5\%$. From all the possible errors affecting our infrared radii and distances, the correct knowledge of p might be the largest remaining uncertainty in the method. Clearly, further work, both theoretical and observational, will be very important to reduce this uncertainty from its current level.

In Table 5, we summarize the discussion of section 4 and present a corresponding error budget in the near-infrared *distance* to an individual Cepheid. The same errors apply to the radii except the uncertainty in the zero point of the surface brightness-color relation. *All possible systematic errors affecting a distance solution in the infrared methods are smaller than 3%*. The total systematic error is computed according to the precepts of Rabinovich (1995), assuming a uniform distribution of each systematic error within the limits given in Table 5. The total random error given in Table 5 reflects the random scatter in our distance and radius solutions presented in Table 3 and refers to those stars which have amplitudes ≥ 0.3 mag in the color curve used in the solution, and very good observational data. These

random uncertainties can be traced back to the uncertainties of the observations used in the solutions. *For a typical large-amplitude Cepheid, we can easily reduce the random error in its radius and distance to 1% if the data used in the analysis are excellent in quality and quantity.* The total error is obtained adding quadratically the total random and the total systematic error. It is found to be about 3% for both near-infrared solutions.

It is worthwhile noting that over a *sample of Cepheids*, the errors introduced by sources a) - d) in Table 5 tend to cancel out while errors e) - h) are truly systematic in the sense that they do affect all the stars in a sample in the same way.

Finally, we want to note that we do expect some sensitivity of the calibrating relations to *metallicity*. It seems likely that the infrared Barnes-Evans method has a lower sensitivity to metallicity than its optical counterpart (whose metallicity sensitivity has been discussed by Gieren, Barnes & Moffett 1993) and does not introduce systematic errors larger than the other ones described in this section, but this has yet to be checked.

Our finding that the total systematic uncertainty of the infrared Cepheid distances is about 3% can be checked with independent methods of Cepheid distance determination, and we will do this in the following section.

5. Comparison of the Infrared Distances to the ZAMS-Fitting Distances of the Parent Clusters and Associations

The ZAMS-fitting distances to the 16 Cepheids of our sample have been reviewed in Gieren & Fouqué (1993). We adopt the same distance moduli here, but we try to estimate an associated uncertainty. Indeed, not all the ZAMS-fitting distances have the same accuracy, depending on the photometric and spectroscopic quality of the cluster/association data, the number of stars involved in the fit, and the confidence in their selection (these stars must be single, unevolved, main-sequence objects). Also systematic errors enter, such as the degree of reliability of the association of the Cepheid with its supposed parent cluster (which depends on the existence of radial velocity information, the projected distance of the star onto the cluster, and is obviously always more doubtful for OB associations than for well-defined clusters), the possible difference in metallicity with the template cluster (Pleiades in all cases), and the true Pleiades distance modulus.

The resulting distances are given in Table 4, together with the ratio of the ZAMS-fitting distance to our adopted distance, and the combined uncertainty of this ratio. The unweighted mean value of the ratio is 1.02 ± 0.04 , which shows that both distance scales are in agreement. However, some values appear to differ significantly from one. In the following, we discuss all cases where the ratio is smaller than 0.9 or larger than 1.1:

SZ Tau: The ZAMS-fitting distance of NGC 1647 (Turner 1992) is of excellent quality, as well as our adopted distance for SZ Tau (from the $V, V - K$ solution). But the star lies at $\sim 2^\circ$ from the cluster center, which corresponds to 19 pc at the distance of the cluster, and its proper motion disagrees with the mean proper motion of the cluster (Geffert et al. 1996). We therefore prefer to exclude SZ Tau as a secure member of the NGC 1647 cluster.

We note that in Turner's study, SZ Tau is the only Cepheid which does not fit the nice period-luminosity relation, which is explained by Turner assuming that the star pulsates in the first overtone mode. However, adopting our shorter distance would equally well put the star onto the relation.

CV Mon and T Mon: The ZAMS-fitting distance of the Mon OB2 association (Turner 1976) is of better quality than the ZAMS-fitting distance of the anonymous cluster which is supposed to contain CV Mon (Turner 1978). From Turner's work (1976, his Fig. 3), it appears that in fact the "CV Mon" cluster is at a compatible distance with the Mon OB2 association. Now, our adopted distance to CV Mon agrees with the ZAMS-fitting distance of Mon OB2. We therefore adopt as the ZAMS-fitting distance of CV Mon the one for Mon OB2 in place of the less accurate ZAMS-fitting distance to the anonymous cluster to which it apparently belongs. Our adopted distance to T Mon (from the $K, J - K$ solution) clearly disagrees with the ZAMS-fitting distance to Mon OB2. We also note that Turner's arguments of membership are very weak. Moreover, the reddening of T Mon ($E(B - V) = 0.209$) is clearly smaller than the reddening of CV Mon ($E(B - V) = 0.714$), which reinforces the conclusion that T Mon lies in the foreground of the Mon OB2 association. We therefore reject T Mon as a member of the Mon OB2 association, and correct the ZAMS-fitting distance of CV Mon to the distance of Mon OB2, without excluding that it may also belong to the loose cluster discovered by van den Bergh (1957).

V340 Nor: The ZAMS-fitting distance to the cluster NGC 6067 (Walker 1985) which contains both V340 Nor and QZ Nor is of excellent quality. This is not the case of our adopted distance to V340 Nor, because the V light curve is poor (which makes the $V, V - K$ solution inaccurate) and the $J - K$ amplitude is small (making the $K, J - K$ solution inaccurate). However, V340 Nor only lies (in projection) at 2' from the center of the cluster, and its radial velocity ($-40.34 \pm 0.39 \text{ km s}^{-1}$ from Bersier et al. 1994) agrees very well with the mean radial velocity of the cluster ($-39.9 \pm 0.16 \text{ km s}^{-1}$ from Mermilliod et al. 1987, from 8 stars). We therefore attribute the discrepancy to the inaccuracy of the infrared Barnes-Evans distance, but still consider V340 Nor as a rather secure member of NGC 6067. The other Cepheid of the cluster, QZ Nor, has an adopted distance in good agreement with the ZAMS-fitting distance of the cluster, although it is located at 18' (in projection) from the cluster center, and has sometimes been rejected as a member of the cluster. In summary, we conclude that both stars are members of NGC 6067, but we do not use the distance ratio of V340 Nor in the final estimate of the agreement of both distance scales.

U Car: This star together with VY Car are generally assigned to the Car OB2 association. Turner's (1988) study of this association does not give details about the quality of the ZAMS-fitting distance. Although the velocity measurements favour membership of both stars ($+1 \pm 2 \text{ km s}^{-1}$ for Car OB2, $+2.0 \text{ km s}^{-1}$ for VY Car, and $+1.7 \text{ km s}^{-1}$ for U Car), we clearly find different distances to both Cepheids: the adopted distance to VY Car is in very good agreement with the ZAMS-fitting distance to Car OB2, while the adopted distance to U Car puts it in the foreground of the association. We therefore reject U Car as an association member.

SV Vul: Turner (1974), quoted in Turner (1979), derived a ZAMS-fitting distance to the

Vul OB1 association, which clearly disagrees with our rather accurate distance determination to SV Vul. We note that published reddening measurements of the Cepheid span a large range ($E(B - V)$ values between 0.41 and 0.58), indicating that the region is complex. We consider that SV Vul is located in the background of the Vul OB1 association, although not as far as the Vul OB2 association (which contains the Cepheid S Vul).

From this discussion, we conclude that four Cepheids in our sample do not belong to their supposed parent cluster or association (SZ Tau, T Mon, U Car and SV Vul). Excluding these four stars as well as the uncertain distance ratio of V340 Nor, and correcting the ZAMS-fitting distance to CV Mon, the mean ratio (ZAMS-fitting distance to infrared Barnes-Evans distance) amounts to 0.987 ± 0.016 (unweighted), or 0.969 ± 0.014 (weighted). This shows that the agreement between both distance scales remain excellent after rejection of non-members. The standard deviation of the ratio is 0.05 (unweighted: 0.053, weighted: 0.047), which means that both distance indicators are accurate to better than 5%, including systematic errors. For instance, a possible systematic error in the Pleiades distance modulus cannot exceed the published standard error (5.57 ± 0.08 from van Leeuwen 1983). The exact accuracy of each method is difficult to estimate; however, the error budget presented in the previous section shows that the infrared surface brightness distances are accurate to 3% in the mean, including random errors and identified systematic errors. We therefore attribute a 4% accuracy in the mean to the ZAMS-fitting distances.

6. Conclusions

We have obtained the radii and distances of 16 supposed galactic cluster Cepheids using our new optical and near-infrared calibrations of the surface brightness method given in Paper I. We confirm the conclusion of Paper I that the radii and distances obtained from both infrared methods (V, V-K and K, J-K) agree very well, and that the typical random uncertainty of the infrared radii and distances is in the order of 2%, while it is generally much larger for the optical V, V-R solutions. Our error budget shows that the systematic errors affecting the infrared radii and distances are in the order of 3%, and that the total uncertainty of the radius and distance of an individual Cepheid is $\sim 3\%$ provided there are very good data and the amplitude of the color curve used in the surface brightness solution exceeds ~ 0.3 mag. The optical solutions are generally affected by both, large systematic and random errors, which are almost certainly due to the variations in microturbulence and gravity during the pulsation cycles of the Cepheids and which manifest themselves clearly in the common plots of the linear and angular diameters versus phase. However, there are a few stars where this problem is much less than for others (like SW Vel, for instance), and it would be interesting to find out the reason for this. We speculate that the amount of systematic deviation of the optical radius and distance from the corresponding infrared value is correlated with the mean microturbulence of the Cepheid, which can vary considerably from one Cepheid to another, even at an almost constant period. This hypothesis could be tested empirically by obtaining accurate spectroscopic mean microturbulence values for the

sample of variables analyzed in this paper.

Our infrared radii agree very well with the infrared radii derived by Laney & Stobie with the maximum likelihood technique, providing further evidence that both techniques are able to determine the radii of Cepheid variables with an accuracy of $\sim 3\%$ (provided the amplitudes of the color and linear displacement curves are not too small).

Our comparison of the adopted infrared distances of the variables to the ZAMS-fitting distances of their supposed host clusters and associations demonstrates that there is excellent agreement between both distance scales. Our accurate distance determinations, together with other available information permit us to identify four non-members among the present sample of Cepheids, while 2 stars (V Cen and BB Sgr) can now be safely included into the list of cluster Cepheids. From the comparison of the infrared surface brightness to the ZAMS-fitting distances it also follows that both methods yield distances accurate to better than 5 percent. Adopting a 3 percent total uncertainty in our infrared distances, as discussed in section 4, the mean accuracy of the ZAMS-fitting distance determinations is about 4 percent, which is somewhat surprising given the difficulties in most of the ZAMS-fitting distance determinations for the present sample of Cepheids. It would of course be interesting to include the northern cluster Cepheids into the comparison, but since the present sample includes ~ 75 percent of the total sample of known or suspected cluster Cepheids, we do not expect any significant change in our conclusions reached in this paper.

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Appendix: Maximum Likelihood vs. Least-Squares Solutions

There is a discussion in Laney & Stobie (1995) about the respective merits of the Maximum Likelihood and Least-Squares fits (hereafter ML and LS) to determine the best values of the Cepheid radii (and distances, in our case). We fully agree with these authors that, *when uncertainties are accurately known*, ML results are superior. However, a small variation of assumed uncertainties may lead to significant variations in the results of the ML method.

In fact, the ML result always lies between the two extreme LS results, namely the direct LS fit assuming no uncertainty in angular diameter values (X axis) and the inverse LS fit, which assumes no uncertainty in linear displacement values (Y axis). Laney & Stobie only compare the ML result to the *direct* LS result, and find significant differences. In fact, we will show below that the ML fit is close to the *inverse* LS fit, because relative accuracies are smaller in angular diameters than in linear displacements. So we agree with Laney & Stobie to reject the direct LS fit, but we prefer to adopt the inverse LS fit rather than the ML fit, because the former has the desirable property of being independent of assumed values of uncertainties.

In order to estimate which of the two variables carries more uncertainty, we must compare the relative accuracy of measurements. As an estimator of linear displacement accuracy, Laney & Stobie propose the ratio of $\sigma(r)$ (uncertainty of radial displacement) to ΔR (difference between maximum and minimum linear radius). The corresponding estimator of angular diameter accuracy would be the angular diameter accuracy $(\sigma(\phi)/\phi)$, multiplied by some average angular diameter, and divided by the difference between maximum and minimum angular diameters. A more convenient, although equivalent, choice of estimators consists of $\sigma(r)/R$, where R is the mean linear radius, and $\sigma(\phi)/\phi$.

In order to compute $\sigma(r)$ in solar radius units, we use the same formula as Laney & Stobie, first introduced by Balona (1977), namely:

$$\sigma(r) = 0.0844 \times P \times \frac{\sigma(RV)}{\sqrt{N}} \quad (A1)$$

where P is the Cepheid period in days, $\sigma(RV)$ is the mean uncertainty of radial velocity measurements in km s^{-1} , and N is the number of radial velocity measurements. The numerical factor 0.0844 is $1.36 \times \frac{86400}{2 \times 696000}$. Resulting values of $\sigma(r)/R$ vary between 0.0016 (CV Mon) and 0.0083 (WZ Sgr), i.e. they are always better than 1%.

Using Eqs. 1-3 of this paper, together with the adopted reddening ratios of Paper I, we derive the following formulae to compute $\sigma(\phi)/\phi$ for each surface brightness – colour relation; we take into account that V and K light curves are independent, but V and R_J , and J and K are not:

$$\frac{\sigma(\phi)}{\phi}(V, V - K) = \ln 10 \times [0.11 \sigma^2(V) + 0.07 \sigma^2(K) + 0.011 \sigma^2[E(B - V)]] \quad (A2)$$

$$\frac{\sigma(\phi)}{\phi}(V, V - R_J) = \ln 10 \times [0.04 \sigma^2(V) + 0.58 \sigma^2(V - R) + 0.007 \sigma^2[E(B - V)]] \quad (A3)$$

$$\frac{\sigma(\phi)}{\phi}(K, J - K) = \ln 10 \times [0.04 \sigma^2(K) + 0.05 \sigma^2(J - K) + 0.002 \sigma^2[E(B - V)]] \quad (A4)$$

For each Cepheid, an estimate of photometric uncertainties has been derived from examination of light curves. $\sigma(V)$ ranges from 0.01 to 0.015, with extreme values of 0.02 (U Car) and 0.03 (V340 Nor). $\sigma(V - R)$ ranges from 0.01 to 0.02. Note that the conversion from the Cousins to Johnson system does not affect the accuracy of the measurements, but may only introduce a systematic error in the derived radius and distance. $\sigma(K)$ and $\sigma(J - K)$ are taken as 0.01, except for EV Sct where infrared light curves are not well sampled, and we estimate both uncertainties to be 0.03.

To estimate $\sigma[E(B - V)]$, we have used Fernie's database of reddenings (Fernie et al. 1995), which gives several measures for each star, to which we added Bersier's (1996) measurements, when available (5 stars). This gives between 7 and 11 values per star, from which we derive an estimate of $\sigma[E(B - V)]$. Resulting values range from 0.025 (S Nor) to 0.072 (SV Vul). For 2 stars, few available values lead to uncertain results: V340 Nor (3 values, $\sigma[E(B - V)] = 0.017$) and QZ Nor (4 values, $\sigma[E(B - V)] = 0.050$).

From these individual uncertainties, we compute the relative accuracy of angular diameter measurements for each colour – magnitude combination. For V , $V - R_J$, results range from 0.019 to 0.038, and the ratios to the corresponding relative accuracy of linear displacements range from 3 to 15. For V , $V - K$, relative accuracies range from 0.011 to 0.024, and ratios from 2 to 13. And for K , $J - K$, relative accuracies range from 0.007 to 0.021, and ratios from 1 to 11. *Therefore, if we only consider measuring uncertainties, we are already justified to use the inverse least-squares solution in all cases.* Now, looking back at the method used to derive angular diameters, any contribution of gravity or microturbulence differences among stars adds to the uncertainty of the angular diameter determination. This is clearly evidenced in the V , $V - R_J$ solution, where the measuring uncertainty alone does not explain the large dispersion observed in the linear displacement vs. angular diameter diagram. This reinforces our decision to use the *inverse LS* solution as the best choice.

Finally, let us say that in cases where this decision may be disputed (typically when the ratio of the relative accuracies is smaller than 2, and assuming that systematic uncertainties in the angular diameter determination may be neglected), the difference between direct and inverse LS solutions (as given in Table 3, columns 4 and 6) is generally smaller than the mean error of the adopted solution, which makes the result insensitive to our choice.

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Figure Captions

- Fig. 1:** *Top panel:* The angular diameters (open circles) of the Cepheid VY Car calculated from the optical V, V-R photometry, plotted against phase. Overplotted (dots) is the linear displacement variation calculated from the integrated radial velocity curve of the Cepheid. *Bottom panel:* Plot of the linear displacements versus the angular diameters. Overplotted is the least-squares inverse fit which takes into account that the angular diameters are more uncertain than the linear displacements.
- Fig. 2:** Same as Fig. 1, for the angular diameters calculated from the infrared K, J-K magnitude-color combination (equ. 3; see text). Note the marked increase in accuracy of the solution, as compared to the optical solution shown in Fig. 1.
- Fig. 3:** Same as Fig. 1 and 2, for the angular diameters calculated from the V, V-K magnitude-color combination. The accuracy of the solution is comparable to the one of the K, J-K solution.
- Fig. 4:** The uncertainties of the radius and distance solutions (normalized to a common number of 30 photometric observations), plotted against the amplitude of the color curve used in the solution, for the three magnitude-color combinations used in this work. The V-K amplitude is large enough even for the lowest-amplitude Cepheids not to affect the accuracy of the solution (bottom panel). In the K, J-K solutions, there is a sharp increase in the uncertainty if the J-K amplitude drops below 0.1 mag. For amplitudes larger than about 0.3 mag, the random uncertainty can be reduced to ~ 1 percent in both infrared solutions.
- Fig. 5:** The ratio of the radii obtained from the V, V-K version of the method to those obtained from K, J-K, plotted against the pulsation period. The mean ratio is very close to unity, independent of period. Note the very low scatter in the diagram. The K, J-K solutions for the three shortest-period Cepheids (open squares) are unreliable due to the very low J-K amplitudes.
- Fig. 6:** The same as Fig. 5, for the distances.
- Fig. 7:** The angular diameters (open circles) and linear displacements (dots) plotted against phase, for the Cepheid U Car and for the K, J-K (top panel) and V, V-K solution (bottom panel). Note the small, but systematic deviation of the angular diameters from the linear displacements in the V, V-K solution near phase 0.3 which is not seen in the K, J-K solution. This effect is very likely due to a slight phase mismatch between the V and K light curves used in the V, V-K solution.
- Fig. 8:** The ratio of the radii obtained from the optical V, V-R solution to the adopted infrared radii (see text), plotted against pulsation period. Note the large scatter, as compared to Fig. 5. Also, the mean ratio is clearly larger than unity.

Fig. 9: Same as Fig. 8, for the distances. Compare to Fig. 6.

Fig. 10: Comparison of the maximum likelihood infrared radii of Laney & Stobie (1995) to our infrared solutions. The three shortest-period and lowest-amplitude Cepheids (open squares) have unreliable maximum likelihood radii. For the remainder of the Cepheids, the agreement of the radii is very good. The mean radius ratio is very close to unity, and the scatter in the diagram is very low.

Fig. 11: Variation of the distance for the Cepheid U Sgr calculated from our K, J-K method as a function of the phase mismatch between the photometric and the radial velocity curve (solid curve). The change in the distance is \sim 1 percent for a change in the phase alignment of 0.01. Also plotted (dotted curve) is the variation of the relative distance uncertainty with the phase mismatch. The minimum of this curve can be used as a criterion to determine the correct phase relation between radial velocities and photometry.

Table 2. Adopted Periods, Color Excesses and p-factors

Cepheid	Period (days)	E(B-V)	$\sigma(E(B-V))$	p
EV Sct	3.09099	0.679	0.045	1.375
SZ Tau	3.14873	0.294	0.035	1.375
QZ Nor	3.7868	0.276	0.050	1.373
CV Mon	5.378793	0.714	0.049	1.368
V Cen	5.49392	0.289	0.033	1.368
BB Sgr	6.63714	0.284	0.040	1.365
U Sgr	6.744925	0.403	0.026	1.365
S Nor	9.754244	0.189	0.025	1.360
V340 Nor	11.287	0.315	0.017:	1.358
VY Car	18.907	0.243	0.067	1.352
RZ Vel	20.402	0.335	0.031	1.351
WZ Sgr	21.8496	0.467	0.070	1.350
SW Vel	23.434	0.349	0.029	1.349
T Mon	27.0197	0.209	0.054	1.347
U Car	38.807	0.283	0.042	1.342
SV Vul	45.04	0.570	0.072	1.340

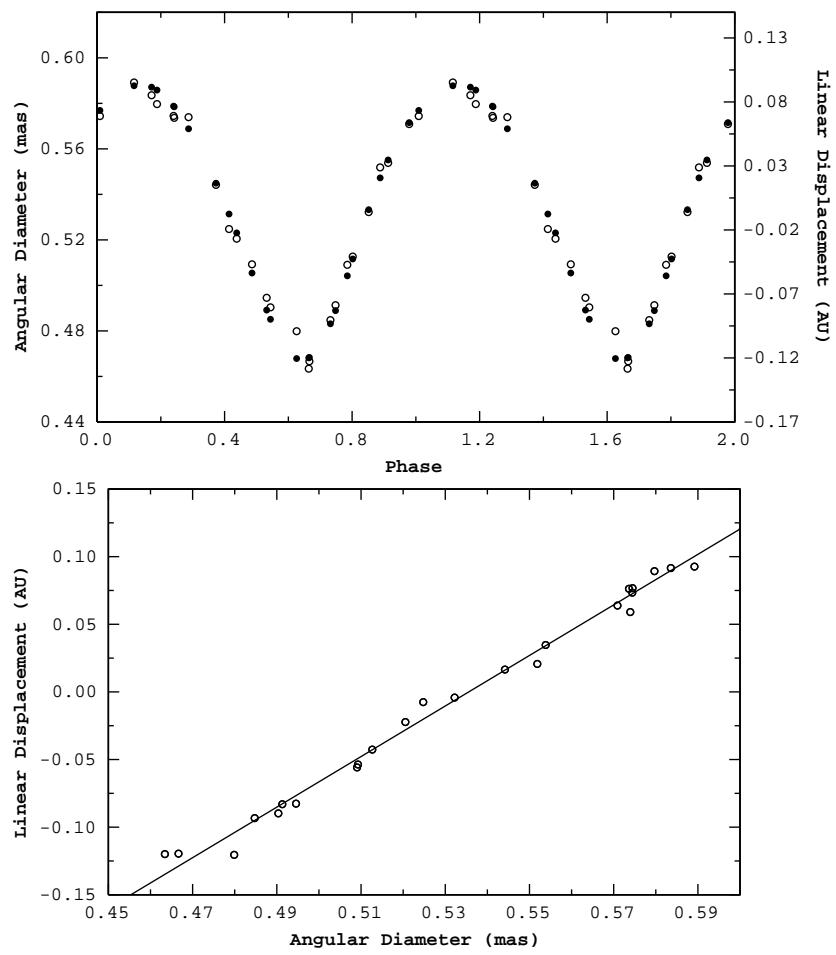


Table 3. Optical and near-infrared Radius and Distance Solutions

Cepheid	Magnitude/ color	Radius [R _☉]	R _{inv-R_{bis}} [σ]	distance [pc]	d _{inv-d_{bis}} [σ]	phase shift	N
EV Sct	V,V-R _J	27.6 ± 5.5	1.7	1344 ± 264	1.7	-0.15	43
	K,J-K	31.6 ± 4.4	0.7	1641 ± 229	0.7	-0.03	12
	V,V-K	32.5 ± 0.5	0.4	1634 ± 25	0.4	-0.04	43
SZ Tau	V,V-R _J	35.9 ± 2.8	1.1	543 ± 43	1.1	0	35
	K,J-K	45.6 ± 4.0	1.3	692 ± 61	1.3	-0.04	17
	V,V-K	27.7 ± 0.5	0.2	415 ± 8	0.4	0	35
QZ Nor	V,V-R _J	56.7 ± 5.1	2.2	2682 ± 241	2.2	-0.03	48
	K,J-K	27.7 ± 2.4	1.1	1221 ± 102	1.1	0	29
	V,V-K	38.5 ± 0.5	0.4	1656 ± 24	0.3	-0.02	49
CV Mon	V,V-R _J	50.3 ± 3.5	1.4	2151 ± 150	1.4	0	54
	K,J-K	42.9 ± 2.0	0.5	1648 ± 76	0.5	-0.03	26
	V,V-K	40.0 ± 0.5	0.2	1507 ± 18	0.3	-0.02	54
V Cen	V,V-R _J	48.8 ± 1.7	0.6	832 ± 29	0.6	-0.05	35
	K,J-K	44.4 ± 1.5	0.8	720 ± 25	0.8	0	30
	V,V-K	45.1 ± 0.5	0.4	726 ± 9	0.4	0	35
BB Sgr	V,V-R _J	49.1 ± 3.9	1.8	764 ± 60	1.8	0	72
	K,J-K	44.4 ± 2.4	0.5	713 ± 37	0.5	0.03	19
	V,V-K	44.4 ± 0.4	0.3	704 ± 7	0.4	0.02	72
U Sgr	V,V-R _J	57.1 ± 3.9	2.3	700 ± 47	2.3	0	81
	K,J-K	50.5 ± 1.5	0.6	621 ± 18	0.6	0	30
	V,V-K	48.8 ± 0.3	0.3	593 ± 4	0.3	0	81
S Nor	V,V-R _J						
	K,J-K	71.1 ± 2.4	0.7	974 ± 32	0.7	0.025	31
	V,V-K	70.9 ± 1.0	0.4	961 ± 12	0.5	0	61
V340 Nor	V,V-R _J						
	K,J-K	82.9 ± 8.3	0.9	2085 ± 212	0.9	0	14
	V,V-K	78.2 ± 5.7	0.6	1951 ± 143	0.6	0.02	30

Table 3: Continued

Cepheid	Magnitude/ color	Radius [R _⊕]	R _{inv} -R _{bis} [σ]	distance [pc]	d _{inv} -d _{bis} [σ]	phase shift	N
VY Car	V,V-R _J	132.0 ± 5.7	0.5	2367 ± 105	0.5	0	23
	K,J-K	110.3 ± 1.8	0.3	1949 ± 32	0.3	0	44
	V,V-K	107.6 ± 2.6	0.2	1869 ± 45	0.2	0	24
RZ Vel	V,V-R _J	130.1 ± 4.4	0.8	1984 ± 66	0.8	0.02	53
	K,J-K	120.8 ± 1.5	0.4	1703 ± 22	0.3	0.02	40
	V,V-K	126.9 ± 3.4	0.3	1753 ± 45	0.3	0.03	53
WZ Sgr	V,V-R _J	160.3 ± 8.0	1.9	2403 ± 123	1.9	0	73
	K,J-K	121.2 ± 2.6	0.4	1794 ± 39	0.4	0	39
	V,V-K	122.5 ± 1.3	0.3	1787 ± 19	0.3	0	73
SU Vul	V,V-R _J	117.0 ± 4.4	0.6	2593 ± 100	0.6	0	27
	K,J-K	119.0 ± 1.7	0.2	2540 ± 36	0.2	0	36
	V,V-K	114.3 ± 2.8	0.2	2394 ± 58	0.2	0	27
T Mon	V,V-R _J	151.2 ± 5.2	0.9	1389 ± 50	0.9	0	51
	K,J-K	133.4 ± 4.1	0.4	1304 ± 40	0.4	0	29
	V,V-K	119.6 ± 1.6	0.3	1151 ± 16	0.3	0	51
U Car	V,V-R _J	163.6 ± 3.4	0.7	1599 ± 35	0.7	0	51
	K,J-K	167.5 ± 3.1	0.2	1636 ± 29	0.2	0.02	33
	V,V-K	175.9 ± 2.7	0.9	1683 ± 26	0.9	0	51
SV Vul	V,V-R _J	224.3 ± 6.9	1.1	2250 ± 68	1.1	0	95
	K,J-K	250.7 ± 8.2	0.5	2918 ± 97	0.5	0	27
	V,V-K	244.5 ± 3.1	0.4	2778 ± 35	0.5	0	95

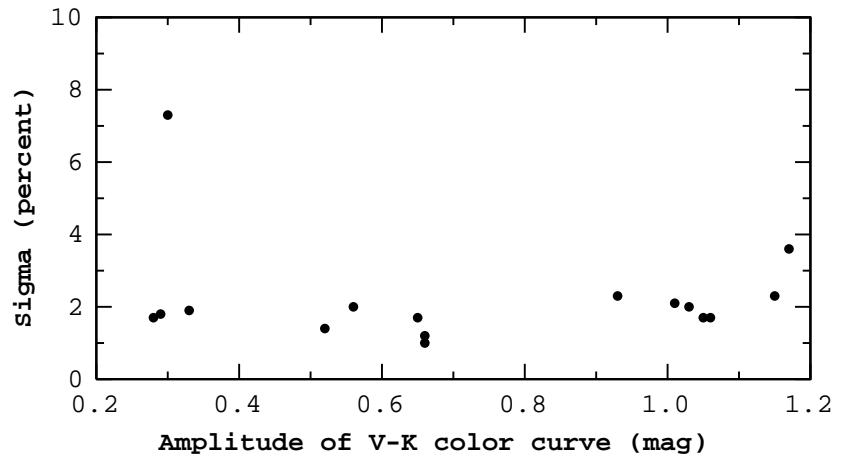
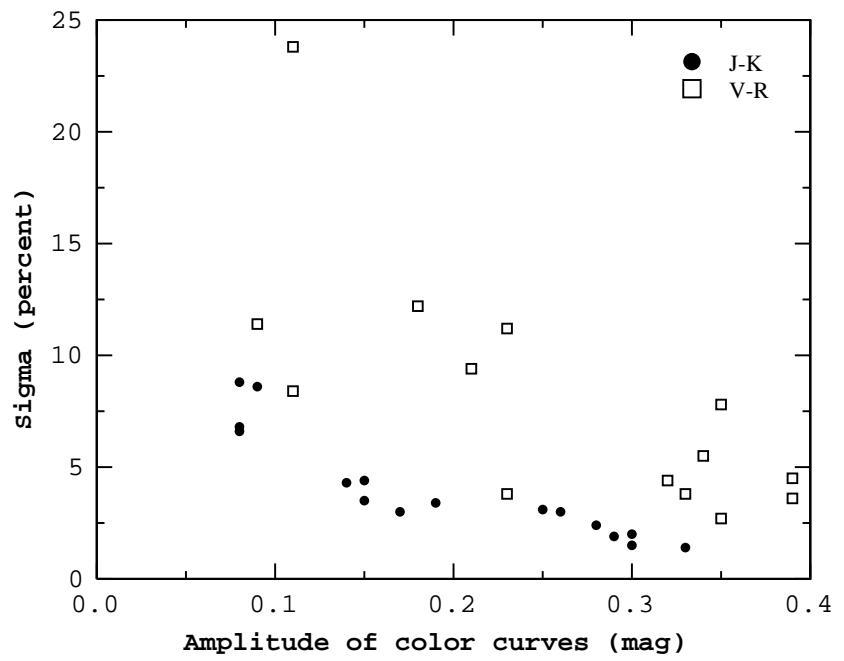


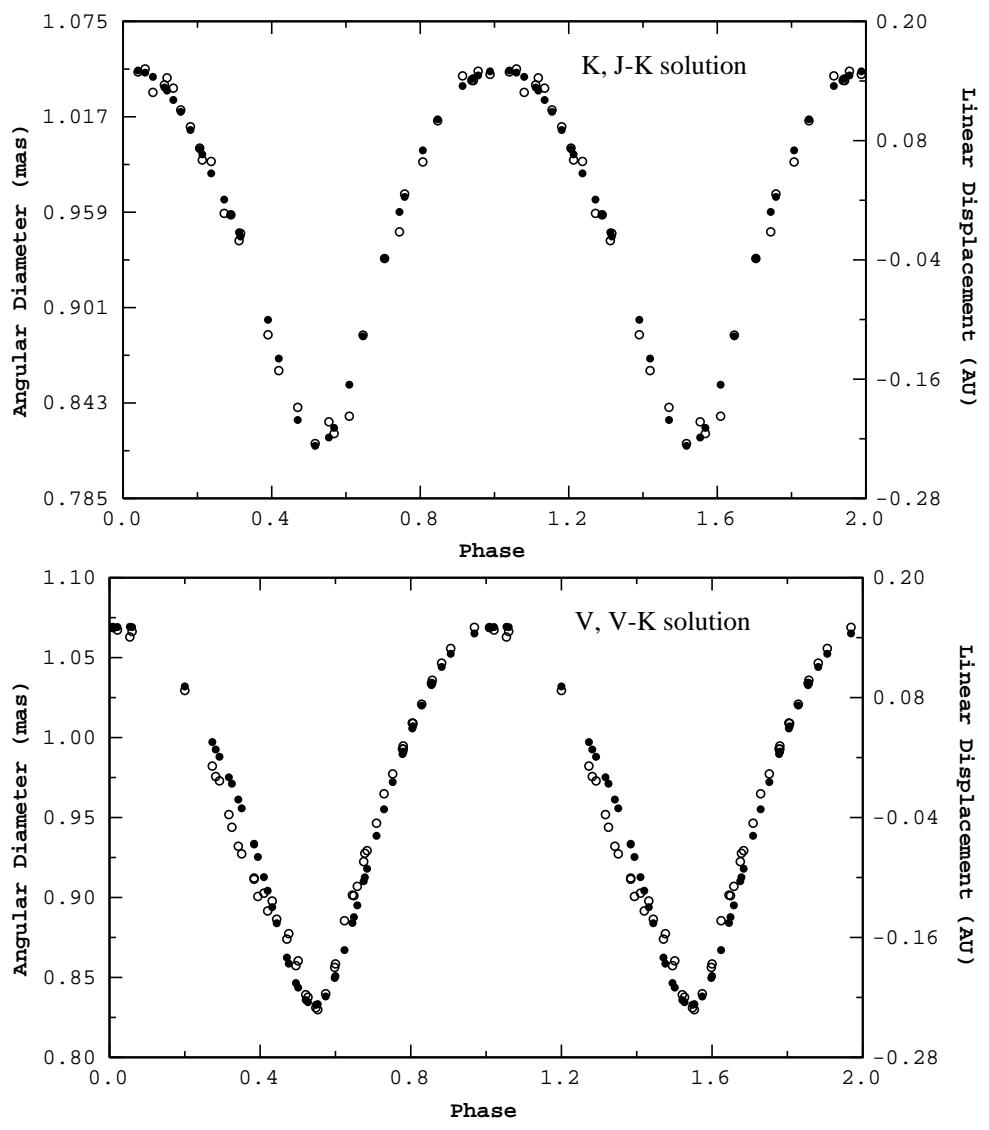
Table 4. Adopted infrared Cepheid radii and distances, and comparison to published values and to distances derived from cluster ZAMS-fitting

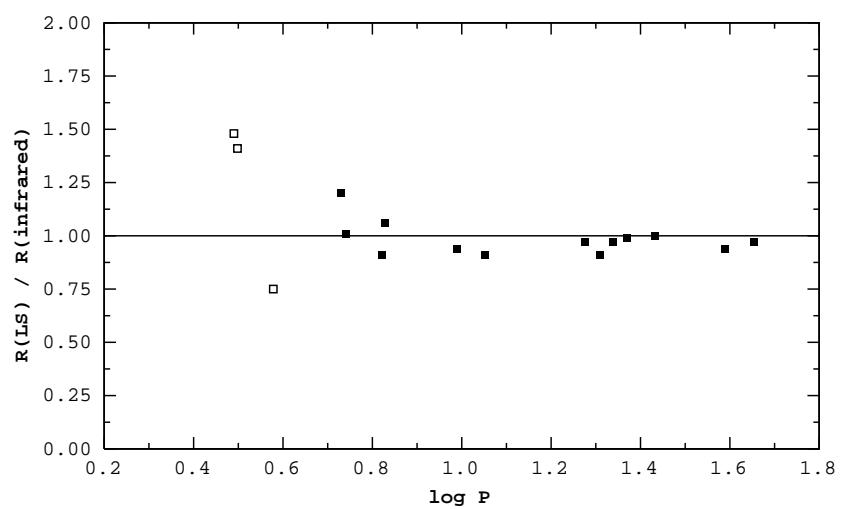
Cepheid	log P [days]	R _{ad} [R _⊕]	R _{V-R} R _{ad}	R _{LS} R _{ad}	R _{GBM} R _{ad}	d _{ad} [pc]	d _{ZAMS} [pc]	d _{ZAMS} d _{ad}
EV Sct	0.4901	32.5 ± 0.5	0.85	1.48:		1634 ± 25	1500 ± 138	0.92 ± 0.09
SZ Tau	0.4981	27.7 ± 0.5	1.30	1.41:	1.37	415 ± 8	545 ± 5	1.31 ± 0.03
QZ Nor	0.5783	38.5 ± 0.5	1.47	0.75:		1656 ± 24	1683 ± 78	1.02 ± 0.05
CV Mon	0.7307	40.2 ± 0.7	1.25	1.20		1514 ± 32	1754 ± 97	1.16 ± 0.07*
V Cen	0.7399	45.0 ± 0.5	1.08	1.01	1.12	725 ± 8	664 ± 24	0.92 ± 0.03
BB Sgr	0.8220	44.4 ± 0.4	1.11	0.91	1.00	704 ± 7	646 ± 6	0.92 ± 0.01
U Sgr	0.8290	48.8 ± 0.3	1.17	1.06	1.17	594 ± 4	617 ± 17	1.04 ± 0.03
S Nor	0.9892	70.9 ± 0.9	–	0.94		963 ± 11	908 ± 13	0.94 ± 0.02
V340 Nor	1.0526	79.7 ± 4.7	–	0.91		1993 ± 119	1683 ± 78	0.84 ± 0.06
VY Car	1.2766	109.4 ± 1.5	1.21	0.97	1.21	1922 ± 38	1923 ± 35	1.00 ± 0.03
RZ Vel	1.3097	121.8 ± 2.3	1.07	0.91	0.99	1713 ± 20	1770 ± 49	1.03 ± 0.03
WZ Sgr	1.3394	122.2 ± 1.2	1.31	0.97	1.24	1788 ± 17	1795 ± 8	1.00 ± 0.01
SW Vel	1.3698	117.7 ± 2.1	0.99	0.99	1.00	2499 ± 65	2512 ± 35	1.01 ± 0.03
T Mon	1.4318	133.4 ± 4.1	1.13	1.00	1.29	1304 ± 40	1600 ± 44	1.23 ± 0.05
U Car	1.5889	167.5 ± 3.1	0.98	0.94	1.01	1636 ± 29	1923 ± 35	1.18 ± 0.03
SV Vul	1.6536	250.7 ± 8.2	0.89	0.97	0.81	2918 ± 97	2312 ± 43	0.79 ± 0.03

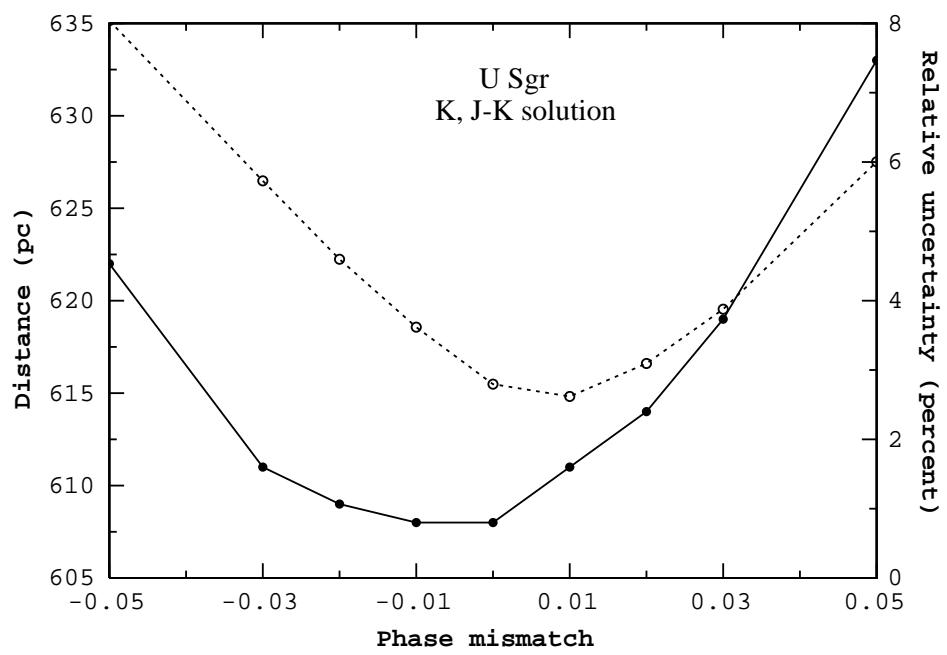
* adopting CV Mon as a member of Mon OB2 (see text), the ratio becomes 1.06 ± 0.04

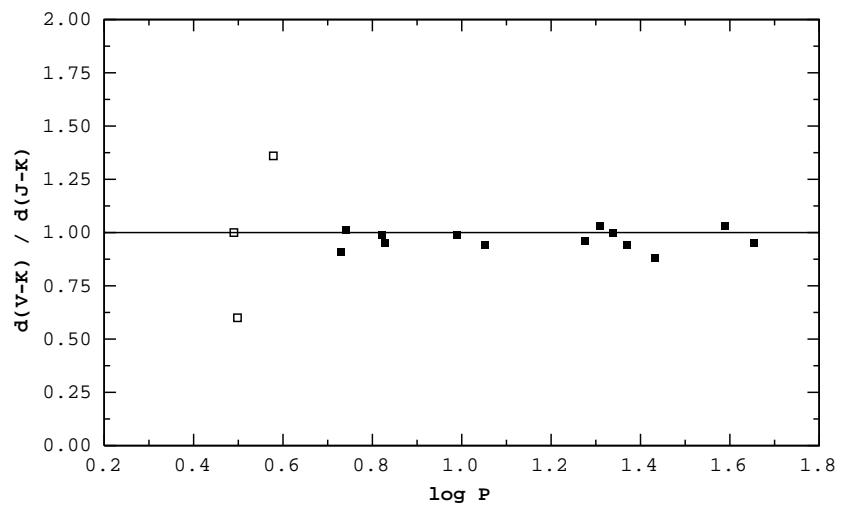
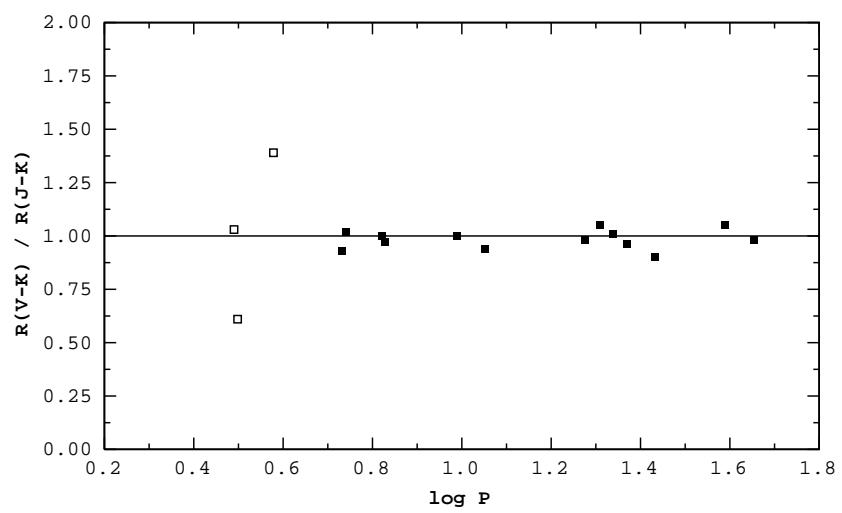
Table 5. Error Budget in the Near-Infrared Distance to an Individual Cepheid

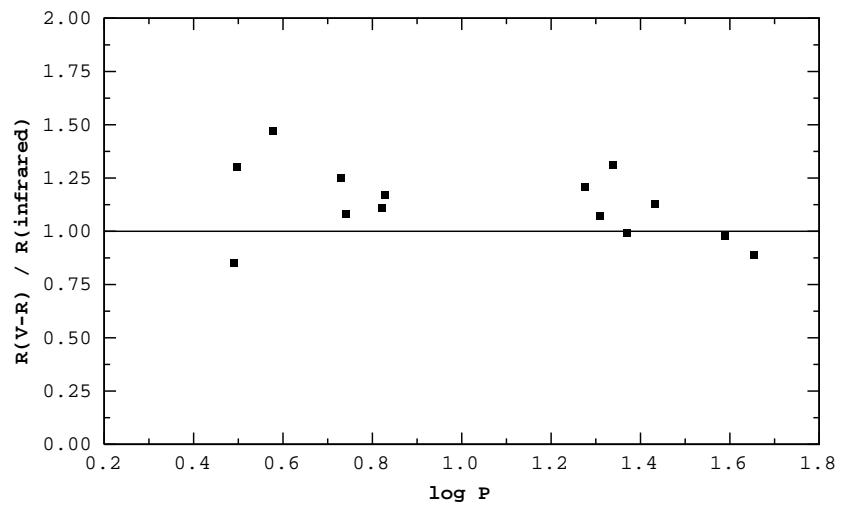
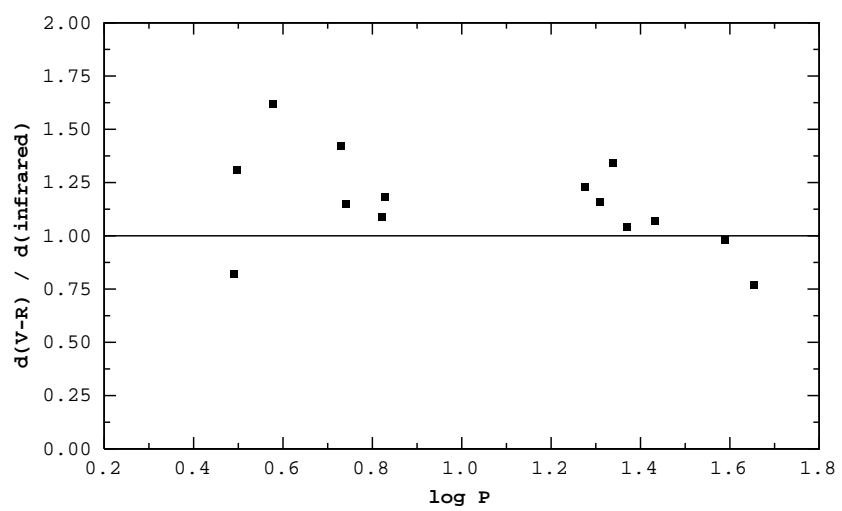
Source of Uncertainty	Systematic Error		Notes
	K,J-K	V,V-K	
a) Error in E(B-V)	$\pm 0.4\%$	$\pm 0.8\%$	
b) Error in Phase Relation between Radial Velocity and Light Curves	$\pm 1.0\%$	$\pm 1.0\%$	Error is ~ 0 in Case of Contemporaneous Data
c) Error in Fourier Fitting of Radial Velocity and K Light Curves	$\pm 0.5\%$	$\pm 0.7\%$	
d) Error in Phase Relation between V and K Light Curves	–	$\pm 2.0\%$	Error is ~ 0 in Case of Contemporaneous Data
e) Error in Zero Point of Surface Brightness-Color Relation	$\pm 2.4\%$	$\pm 2.2\%$	Change of 1σ assumed (see Paper I)
f) Error in Slope of Surface Brightness-Color Relation	$\pm 2.3\%$	$\pm 2.8\%$	Change of 1σ assumed (see Paper I)
g) Error in Adopted Least-Squares Fit (Inverse Solution)	$+1.0\%$	$+1.0\%$	
h) Error in projection factor p	$\pm 2.5\%$	$\pm 2.5\%$	
i) Total Systematic Error	$\pm 2.4\%$	$\pm 2.9\%$	
j) Total Random Error	$\pm 1.5\%$	$\pm 1.0\%$	for Stars with Color Curve Amplitudes $\gtrsim 0.3$ mag
k) Total Error	$\pm 2.9\%$	$\pm 3.1\%$	











Very accurate Distances and Radii of Open Cluster Cepheids from a Near-Infrared Surface Brightness Technique

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